Volatility

Annualized Standard Deviation of the change in price/ value of a financial security.

How do we predict/ estimate this?

- Historical/ Sample Volatility measures
- Geometric Brownian Motion Model
- Poisson Jump Diffusion Model
- ARCH/ GARCH Models
- Stochastic Volatility
- Implied volatility from options/ derivatives

Computing volatility from historical series of actual prices:

- ➔ Prices of an asset at (T+1) time points
  $\{ P_t, t = 0, 1, 2, \ldots, T \}$

- ➔ Returns of the asset for T time periods
  $R_t = log(p_t / p_{t-1}), t = 1, 2, \ldots, T$

This is assumed to be covariance stationary, i.e. it is assumed to have a finite variance.

- ➔ $\{ R_t \}$ assumed covariance stationary with
  $\sigma = \sqrt{var\ (R_t)} = \sqrt{E[(R_t - E[R_t])^2]}$

  With unbiased sample estimate:

  $\sigma' = \sqrt{(1/T - 1) * \sum_{t=1}^{T} (R_t - R')^2}$, with $R' = (1/T) \sum_{1}^{T} R_t$

- ➔ Annualized Values
  $Vol' = \sqrt{12}\,\sigma'$ for monthly prices

Prediction Methods based on Historical Volatility

Def:- For a time period T, the sample volatility σ'_t = sample standard deviation of period t returns.

Historical Average:

$\sigma^2_{t+1} = 1/t * (\sum_{1}^{t} \sigma'^2_j)$ --- takes into account all the historical values

Simple Moving Average:

$\sigma^2_{t+1} = 1/m * (\sum_{0}^{m-1} \sigma'^2_{t-j})$ --- takes into account last 'm' single period sample estimates.

Exponential Moving Average:

$\sigma^2_{t+1} = (1-\beta)\,\sigma'^2_t + \beta\sigma'^2_t$ --- basically just a convex optimization of the Historical Average and therefore makes use of all available data.

[also $0 <= \beta <= 1$]

Exponential Weighted Moving Average:

$\sigma^2_{t+1} = 1/m * (\sum_{0}^{m-1} \beta^j \sigma'^2_{t-j}) / [\sum_{0}^{m-1} \beta^j]$ --- takes into account last 'm' single period

Geometric Brownian Motion

For {S(t)} the price of a security/ portfolio at time t:

$dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$

Where,

σ = volatility of a security's price

μ = mean return per unit time

$\sigma S(t)$ = infinitesimal increment in the price

$dW(t)$ = infinitesimal increment in Standard Brownian Motion/ Wiener Process

Sample the data from the process:

- Prices: {S(t), t = $t_0, t_1, .. t_n$ }
- Returns: { $R_j = log\,[S(t_j)/ S(t_{j-1})]$, j = 1...n}

Where, $R_j = N(\mu \Delta_j, \sigma^2 \Delta_j)$ and $\Delta_j = t_j - t_{j-1}$

({log[S(t)]} is Brownian Motion with Drift μ and Volatility σ^2)

Then apply MLE to these samples! Δ_j can vary.

If we increase the sampling rate of the process, it gives better estimates for some parameters with lower errors.

The Garman-Klass Estimator:
Done where we have much more information than just the opening and closing prices. Let's say that we have open, high and low prices over the periods along with the closing prices.

This estimator assumes that the mean(μ) = 0 i.e. we are only concerned with the volatility of the data. Let the daily increments be denoted by $\Delta_j = 1$ and let 'f' denote the fraction of the day prior to the market open.

Then
$C_j =$ log[S(t_j)]
$O_j =$ log[S($t_{j-1} + f$)]
$H_j =$ max[S(t_j)]
$L_j =$ min[S(t_j)]

So the difference between 2 closes can be modeled as:

$C_1 - C_0 = N(0, \sigma^2)$ ---1
We also know that:
$(C_1 - C_0)^2 = \sigma^2 \chi_1$, when 1 is squared, we get a multiple of the chi-squared distribution.

E[χ_1] = 1 and Var[χ_1]=2

Similar estimates can be derived between 2 closes and etc…

Next___ Poisson Jump Process!